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Computational Inverse Technique for Material Characterization of Functionally Graded Materials

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A computational inverse technique is presented for characterizing the material property of functionally graded material (FGM) plates, using the dynamic displacement response on the surface of the plate as input data. A modified hybrid numerical method is used as the forward solver to calculate the dynamic displacement response on the surface of the plate for given material property varying continuously in the thickness direction. A uniform crossover micro-genetic algorithm (uniform μGA) is employed as the inverse operator to determine the distribution of the material property in the thickness direction of the FGM plate. Examples are presented to demonstrate this inverse technique for material characterization of FGM plates. The sensitivity and stability to noise contamination in the input displacement response data are also investigated in detail. It is found that the present inverse procedure is very robust for determining the material property distribution in the thickness direction of FGM plates.

Nomenclature

a	=	mean of the noise
b	=	standard deviation of the noise
$\boldsymbol{c}$	=	matrix of elastic constants
$\boldsymbol{d}$	=	displacement vector
$\bar{\boldsymbol{d}}$	=	nodal displacement amplitude vector
$\tilde{\boldsymbol{d}}$	=	Fourier transformation of nodal displacement vector
E	=	Young's modulus of elasticity
err(p)	=	square error between the actual and calculated value
$\boldsymbol{F}, \boldsymbol{T}$	=	external force vector
$\tilde{\boldsymbol{F}}, \tilde{\boldsymbol{T}}$	=	Fourier transformation of external force vector
G	=	shear modulus
H	=	thickness of the structure
h_n	=	thickness of the nth layer element
K	=	bulk modulus
$\boldsymbol{K}$	=	stiffness matrix
k_x	=	wave numbers in x directions
M	=	mass matrix
$\boldsymbol{p}$	=	vector of material property
$\boldsymbol{p}_{\min}, \boldsymbol{p}_{\max}$	=	boundary values of the material property
t	=	time
u	=	displacement components
V_p	=	volume fraction
ν	=	Poisson's ratio
ρ	=	mass density

Subscripts

f	=	external force
$1, 2, \ldots, n$	=	number of layer elements

Superscripts

l, m, u	=	lower, middle, and upper surface of a layer element for plate, respectively

Introduction

FUNCTIONALLY graded materials (FGMs) have been proposed for the thermal protection of propulsion systems and airframes of space planes and hypersonic aircraft. Many techniques[1–3] have been developed for fabricating various types of FGMs, in which the materials properties change continuously in the thickness direction in a functionally desirable fashion. FGMs can be used not only in the thermal-protection systems of space planes but also in electrical and chemical engineering, as well as many other engineered systems.

Effective use of FGM relies on a precise knowledge of the material property of the constituent materials; hence, their property evaluation has been one of the focuses of research. Elastic constants of materials are conventionally determined using properly designed specimens for tensile and shear testing. The alternative is to use phase velocities of ultrasonic bulk waves. The Christoffel equation was adopted to establish the relationship between material properties and bulk wave velocity. The full set of elastic constants of anisotropic materials can be determined from the velocity measurements in different directions.[4–6] The elastic constants of orthotropic cylindrical composite shells were characterized using the method of Bayesian estimation based on their natural frequencies obtained from a free–free configuration model.[7] Material properties of laminated polymeric composites are reconstructed using the numerical–experimental method from the experimental results of the structure response.[8,9] All of these techniques can have difficulties or problems in characterizing material properties of FGMs because they are varying in the thickness direction. Currently, there is no simple way to measure nondestructively the material properties of FGMs. The development of an effective and nondestructive technique to achieve such a purpose is, therefore, desirable. Utilizing elastic waves for such a purpose can be promising because we have a reasonably good knowledge about elastic waves propagating in FGM plates.[10–15] Numerical models of FGM plates have been developed to solve forward problems that relate the material property to elastic wave fields. Thus, if a set of reasonably accurate measurements of the displacement response is available, the material properties of the FGM plate may be identified by solving a formulated inverse problem.

In this paper, an inverse technique is presented to obtain the material property and its distribution in FGM plates from the dynamic displacement response on the surface of the plate. In this work, the

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input data used for inverse procedure are the time history of displacements on the surface of an FGM plate, which can be easily measured using conventional experimental techniques.

For a successful implementation of an inverse procedure, there must first be an efficient and accurate forward solver for establishing the relationship between the dynamic displacement response and the material property of an FGM plate subjected to a dynamic excitation. A modified hybrid numerical method (HNM)[14] is employed for the forward calculation to obtain the displacement response for a given material property of an FGM plate. The modified HNM is developed based on the HNM[16–19] to accommodate a linear variation of material properties in an element in the thickness direction. The modified HNM can further reduce the number of elements and can obtain effectively more accurate results.

Gradient-based optimal methods have been commonly employed as the inverse operators in most cases of inverse determination of material properties of structures. For instance, Liu et al.[14] suggested a basic inversion procedure for material characterization using traditional optimization methods. However, as mentioned by Chu et al.,[6] these gradient-based optimal methods are highly dependent on the initial guess and entrapment at a local minimum that are not close to the global minimum. Furthermore, the gradient-based operators are sensitive to the noise (experimental errors). On the contrary, genetic algorithm (GA) uses simple coding technique and artificial genetic process to produce the optimal selection from the more complex global problems. Because GA is not a gradient-based search technique, no initial guesses are required. Moreover, GA begins from a set of points in the search space rather than a single point. All of these features tend to make GA applicable in a wide variety of fields, such as optimization, machine learning, and parallel techniques. Goldberg[20] summarized that GAs are different from those normal optimization and search procedures in four ways:

1) GAs work with a coding of the parameter set, not the parameters themselves.

2) GAs search from a population of points, not a single point.

3) GAs use only objective function information, not derivative or other auxiliary knowledge.

4) GAs use probabilistic transition rules, not deterministic rules. Balasubramaniam and Bao[21] have given a very detailed comparison between GA and normal optimization and search procedures, and employ GA to reconstruct material stiffness properties of a composite from obliquely incident ultrasonic bulk wave data. GA has also been used for optimizing design of composite structures.[22–24] Because of the advantages of GA, it is employed in this paper as the inversion operator, which controls the running of the forward solver.

The FGMs are usually microscopically heterogeneous and are typically made from two isotropic components, such as metals and ceramics. The property of the combined materials can be obtained using methods of rule of mixture derived from the micromechanics by using the properties of the components. Because the material property of the components is usually available, the volume fraction and its variation in the thickness direction of the FGM plate are the key parameters in material property characterization of FGMs. As long as the volume fraction is known, the material property can be obtained easily using the rule of mixture. Therefore, the characterization of the material property of an FGM is actually equated to the characterization of volume fractions.

The present characterization process is examined for two FGM plates. Parameters in the uniform μGA are studied and properly tuned for the material characterization of FGM. The sensitivity and stability to noise contamination in the input displacement response data are also investigated and reported in detail.

Process for Material Characterization

An inverse process for material characterization of an FGM plate is outlined in Fig. 1. Here the modified HNM is used as the forward calculation technique, and the uniform μGA is employed as the inverse operator. In a uniform μGA run, each individual chromosome represents a candidate combination of the parameters (volume fractions). For each candidate combination, the material property of the FGM can be obtained using the rule of mixture, and the dynamic displacement responses on the surface of the plate can be calculated using the forward technique. Then the fitness function, which minimizes the sum of squares of difference between calculated and measured response, can determine the trial volume fractions being chosen as a future parent. The stopping criterion is imposed to limit each uniform μGA run to a maximum number of generations. The detail of each box will be given in the following sections.

Forward Calculation Technique

In a forward calculation, one needs to calculate the wave fields in an FGM plate subjected to an incident wave for given material properties and their variation though the thickness. Both efficiency

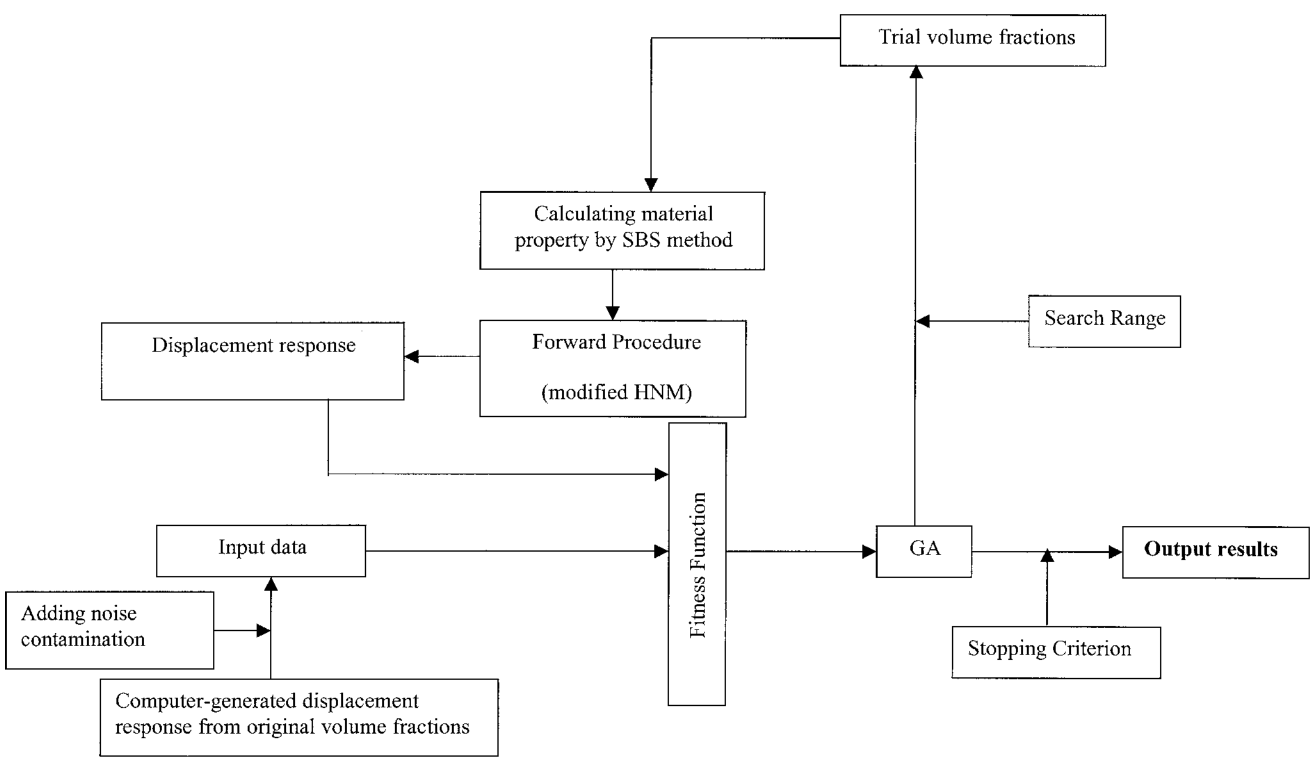

Fig. 1 Flowchart of the present inverse procedure.

and accuracy are very important because forward computations may have to be carried out thousands of times in the later inverse process. The modified HNM[14] used in this work allows a linear variation of material properties in the element in the thickness direction. This is to reduce the number of elements needed to model the material variation of FGM plates. A brief description of the formulation of the modified HNM is given as follows.

Consider an FGM plate with varying material properties in the thickness direction. The thickness of the plate is denoted by H. The plate is subjected a line load uniformly distributed along the circumferential direction, and the load is independent of the y direction, thus the problem in hand is reduced to a two-dimensional problem. The plate is divided into N layered elements. The thickness of the nth element is h_n. The elastic coefficient matrix and the mass density on the lower and upper surface of the nth element are denoted by $c_n^l = (c_{ij})_n^l$, $i, j = 1, \ldots, 6$, ρ_n^l, $c_n^u = (c_{ij})_n^u$, and ρ_n^u, respectively, as shown in Fig. 2. In the nth element, we assume that the material properties change linearly in the thickness direction:

$$(c_{ij})_n = \left[\left(c_{ij}^u\right)_n - \left(c_{ij}^l\right)_n\right](z/h_n) + \left(c_{ij}^l\right)_n \tag{1}$$

$$\rho_n = \left(\rho_n^u - \rho_n^l\right)(z/h_n) + \rho_n^l \tag{2}$$

A set of approximate partial differential equations for an element is obtained using the principle of virtual work. Assembling the matrices of adjacent elements, we obtain the dynamic equilibrium equation of the whole FGM plate[14]:

$$\boldsymbol{F}(x,t) = \boldsymbol{K}\boldsymbol{d}(x,t) + \boldsymbol{M}\ddot{\boldsymbol{d}}(x,t) \tag{3}$$

where $\boldsymbol{F}$ is the external force vector acting on the nodal planes that divide the plate into layered elements and $\boldsymbol{d}$ is the displacement vector on the nodal planes. The matrix $\boldsymbol{K}$ is a differential operator matrix for the plate.

We introduce the Fourier transformations with respect to the axial coordinate x in the form

$$\tilde{\boldsymbol{d}}(k,t) = \int_{-\infty}^{+\infty} \boldsymbol{d}(x,t)e^{ikx}\,\mathrm{d}x \tag{4}$$

where real transformation parameters k_x are the wave numbers corresponding to the axial coordinates x. The application of Eq. (4) to Eq. (3) leads to a set of dynamic equilibrium equations for the FGM plate:

$$\tilde{\boldsymbol{F}} = \boldsymbol{M}\ddot{\tilde{\boldsymbol{d}}} + \boldsymbol{K}\tilde{\boldsymbol{d}} \tag{5}$$

where $\tilde{\boldsymbol{F}}$, $\ddot{\tilde{\boldsymbol{d}}}$, and $\tilde{d}$ are the Fourier transformations of $\boldsymbol{F}$, $\ddot{\boldsymbol{d}}$, and $\boldsymbol{d}$, respectively.

When the modal analysis is used, the displacement vector $\tilde{\boldsymbol{d}}$ in the Fourier transformation domain can be obtained. Finally, the displacement response in the space–time domain can be obtained using the inverse Fourier transformation.[16–18]

In this work, the incident excitation wave to the plate is assumed to be a vertical line load acting at $x = 0$ on the upper surface of the plate. The line load is independent of the y axis, but is a function of t as

$$f(t) = \begin{cases} \sin(\omega_f t), & 0 \le t \le 2\pi/\omega_f \\ 0, & \text{others} \end{cases} \tag{6}$$

Characterization is based on an overdetermined data set, which can be the time history of the displacement response at one receiver point on the upper surface of the FGM plate. It has been found that

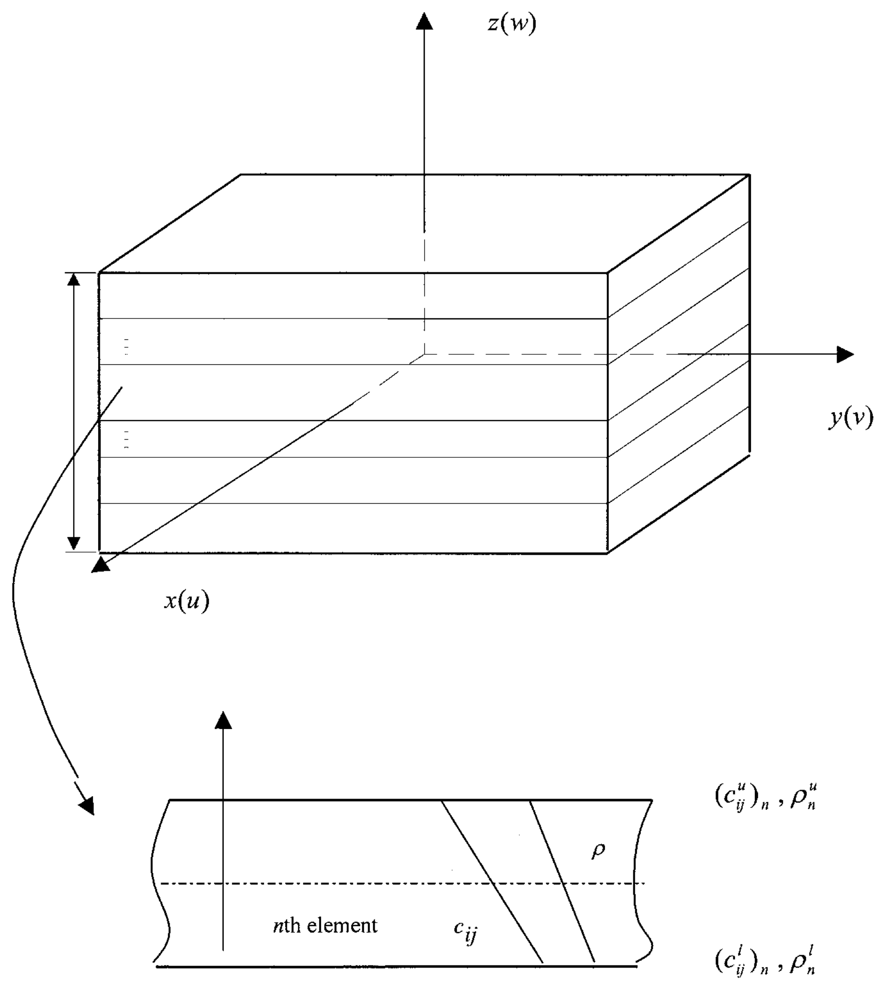

Fig. 2 FGM plate divided into N layer elements in the thickness direction.

using the modified HNM, the response calculation is very fast. A single run takes only 30 s of CPU time at an HP workstation. This high efficiency paves the way for the following inverse procedure of material characterization of the FGM plate.

Heretofore, we have described the forward calculation, that is, using the known material property to obtain the displacement. It is a very complex process and cannot be described in one or two state equations. Now, for conventional illustration in the inverse procedure, we give the following notation for the forward calculation, to define the calculated displacement response for given material property $\boldsymbol{p}$:

$$u^c = \chi(\boldsymbol{p}) \tag{7}$$

Inverse Procedure

Consider an FGM made of two components of isotropic materials. It is assumed that the material properties of the components are known. The material property of an FGM can be obtained using the rule of mixture from the volume fraction of components. When the forward problem solver is used, the displacement response of the FGM plate can be obtained for a set of material properties varying through the thickness of the plate, which is obtained using assumed volume fraction for the two components. The obtained response is, in general, different from those measured from an actual FGM plate. The inverse procedure can then be formulated by an optimal control problem, which minimizes the sum of squares of difference between calculated and measured response. The optimization problem can be stated as follows[25]:

Find $\boldsymbol{p}$ that minimizes

$$\mathrm{err}(\boldsymbol{p}) = \sum_{i=1}^{M} g_i(\boldsymbol{p}, u_i) \tag{8}$$

subjected to

$$\boldsymbol{p}_{\min} \le \boldsymbol{p} \le \boldsymbol{p}_{\max} \tag{9}$$

$$g_i(\boldsymbol{p}, u_i) = \left(u_i^m - u_i^c\right)^2 \tag{10}$$

where $\boldsymbol{p}$ is the control variable that represents the trial parameters that are the volume fractions in this paper. M is the number of times (or locations) when (or where) the displacement response is sampled. Note that the number of sample points should be larger than the number of parameters to be identified. Here, g_i is the contribution of the ith sample point to the total objective function and u_i^m is the displacement response of ith sample point obtained from experimental measurements for FGM plates. We utilize computer-generated displacement response based on the actual volume fraction of the FGM plate. The displacement response data set can be created conveniently, using the displacements either at difference positions or at difference time sequences. Also, u_i^c is the displacement response of the ith sample point obtained from the forward calculation using the trial control variables, as defined in Eq. (7), and $\boldsymbol{p}_{\min}$ and $\boldsymbol{p}_{\max}$ are the lower and upper bound of the control variables, respectively.

In a GA run, each individual chromosome represents a candidate combination of reconstructed parameters (the control variables). For each candidate combination, forward calculation has to be performed to obtain u_i^c. These calculated displacement readings are used to obtain the fitness value of the candidate combination. The fitness value, which is defined using Eq. (8), will determine the probability of the candidate being chosen as a future parent. A FORTRAN subprogram for the forward calculation using the modified HNM is developed and interfaced with the GA main program.

Uniform μGA

The μGA was proposed by Krishnakumar.[26] A μGA program starts with a random, very small population of chromosomes (usually five or six individuals). The population evolves in normal GA fashion, such as performing reproduction and crossover operations. After a few generations, the population converges. (A converge criteria should be preset.) At this point, a new population is randomly generated while keeping the best individual from the previous converged generation, and the evolution process restarts. This process will repeat until the preset stopping criterion is met.

A great difference of μGA from a simple GA is that a μGA program has no mutation operator. With the introduction of the technique of micropopulation, the μGA guarantees it is robust in a different way: Whenever the micropopulation is reborn, new chromosomes are randomly generated, and new genetic information keeps flowing in. Krishnakumar's study[26] has pointed out that a μGA can avoid premature convergence and demonstrates faster convergence to the near optimal region than a simple GA does for the multimodal problems.

Developed by Syswerda,[27] the uniform crossover operator generally works better than one-point and two-point crossover. Following the reproduction process, in which pairs of chromosomes have been chosen for mating and stored in the mating pool, the uniform crossover operator proceeds. For a bit of two mated chromosomes of the same position, a random number is generated to compare with a preset crossover possibility. If the random number is larger than the crossover possibility, the crossover operator swaps the two bits of the mated chromosome. On the other hand, if the random number is smaller, the two chromosomes remain unchanged, and the crossover operation on this bit is over. This crossover operation is performed on every bit of the mated chromosomes in sequence. When the crossover operation completes, two new chromosomes are created for the next GA operations.

It has been found by Krishnakumar[26] that a uniform crossover uniform μGA avoids premature convergence and fast converge to the near-optimal range more than a simple GA for the multimodal problem. The uniform μGA needs a very small population size, usually a population size of five is roughly optimal. According to Carroll,[28] the uniform μGA has two additional benefits: "(i) there is no need to fiddle with GA operators such as mutation probabilities, and (ii) it is able to handle a loosely ordered order-3 deceptive function which the more traditional GA methods and the single-point crossover μGA were not able to optimize." Because new chromosomes with new genetic information keep flowing in when the micropopulation is reborn, the uniform μGA shows better robustness. For these reasons, the uniform μGA is used in this study as the inverse operator for material characterization of FGM plates.

Rule of Mixture

The material property of the FGM can be obtained using the method of rule of mixture derived from micromechanics by using the material properties of the matrix and inclusion for a given volume fraction. Here a step-by-step (SBS) method proposed by Liu[29] is used to predict the material property of FGM for a given volume fraction. In the SBS method, the composite material is composed through a hypothetical process in which one component is treated as matrix and another is treated as inclusion; the inclusion is mixed one-by-one into matrix. The material properties of the composite materials are obtained SBS using the cylinder model and sphere model[30] for fiber and particle-reinforced materials, respectively.

Consider a composite made through particle mixture with a volume fraction of V_P for inclusion; a sufficiently small volume fraction at one step is denoted by V_{P1}. After n steps of mixing, we can obtain the property at the nth step as the final property of composition:

$$K = K_n = K_{n-1} + \frac{(3K_{n-1} + 4G_{n-1})(K_P - K_{n-1})V_{P1}}{(3K_{n-1} + 4G_{n-1}) + 3(K_P - K_{n-1})} \tag{11}$$

$$G = G_n = G_{n-1} + \frac{5(3K_{n-1} + 4G_{n-1})(G_{n-1} - G_P)G_{n-1}V_{P1}}{9K_{n-1}G_{n-1} + 8G_{n-1}^2 + 6(K_{n-1} + 2G_{n-1})G_P} \tag{12}$$

$$E = \frac{9KG}{3K + G} \tag{13}$$

$$\nu = \frac{E}{2G} - 1 \tag{14}$$

The material properties of the inclusion are denoted with subscript P. Here n is given by

$$n = \frac{\ln(1 - V_P)}{\ln(1 - V_{P1})} \tag{15}$$

Furthermore, the mass density is obtained as

$$\rho = \rho_m + (\rho_P - \rho_m)V_P \tag{16}$$

where ρ_m is the mass density of the matrix.

Note that, if the values of V_P in each element surface are found, the material property at the element surface could be obtained using Eqs. (13–16). The variation of the material property in the thickness direction can also be determined by assuming linear variation. Therefore, the volume fractions in each element's surface are chosen as the parameters to be reconstructed for characterizing the material properties of the FGM. If the plate is divided into m layered elements, then there are a total of $m + 1$ discrete values of V_P at the element interfaces and the upper and lower surfaces of the plate.

Results and Discussion

Two different FGM systems with known volume fractions from literature are reconstructed. The first one is an SiC–C FGM plate, which is developed by combining materials SiC and C using a chemical vapor deposition technique.[1] The actual volume fractions and the variation of the SiC–C FGM can be obtained using the method given by Kerner.[31] The material properties of the SiC and C monolith are given in Table 1; the material C is taken as the inclusion material. Another FGM is composed of stainless steel and silicon nitride (SS–SN); the material properties for SS–SN are listed in Table 2 (Ref. 32). The SN is considered as the inclusion material.

Both the noise-free and noise-contaminated displacement response is used for the characterization of the volume fractions. Gauss noise of various levels is directly added to the computer-generated displacements. A vector of pseudorandom number is generated from a Gauss distribution with mean a and standard deviation b using the Box–Muller method (see Ref. 33). In this work, the mean a is set to zero, and the standard deviation b is defined as[34]

$$b = p_e \times \left[\frac{1}{M}\sum_{i=1}^{M}\left(u_i^m\right)^2\right]^{0.5} \tag{17}$$

where u_i^m is the computer-generated displacement reading at the ith sample point and p_e is the value to control the level of the noise contamination, for example, $p_e = 0.05$ means 5% noise. To investigate the sensitivity and stability of present inverse procedure to noise, three noise levels: 2, 5, and 10% are considered.

First consider the SiC–C FGM plate. Parameters in uniform μGA and optimal search range of this material characterization of a FGM will be found through the study of this case. Furthermore, the sensitivity and stability to noise contamination are investigated. The SiC–C FGM plate is divided into five elements, therefore, six parameters (volume fractions) need be reconstructed. The actual values of the volume fractions are listed in the third column in Table 3.

Table 1 Material properties of SiC and C monolith materials

Material constant	E, GPa	ν	ρ, g/cm^3
SiC	320	0.3	3.22
C	28	0.3	1.78

Table 2 Material properties of SS and SN monolith materials

Material	E, GPa	ν	ρ, kg/m^3
SS	207.82	0.3177	8166
SN	322.4	0.24	2370

Table 3 Detailed results of influence of p_{cross} in uniform μGA

p_{cross}	Generation number of search solution	Final value of fitness function	Maximum deviation of individual parameter, %
0.1	493	1.5×10^{-2}	11.5
0.3	496	5.0×10^{-3}	7.5
0.5	326	2.0×10^{-3}	4.5
0.6	367	2.0×10^{-3}	3.8
0.9	448	1.0×10^{-2}	8.6

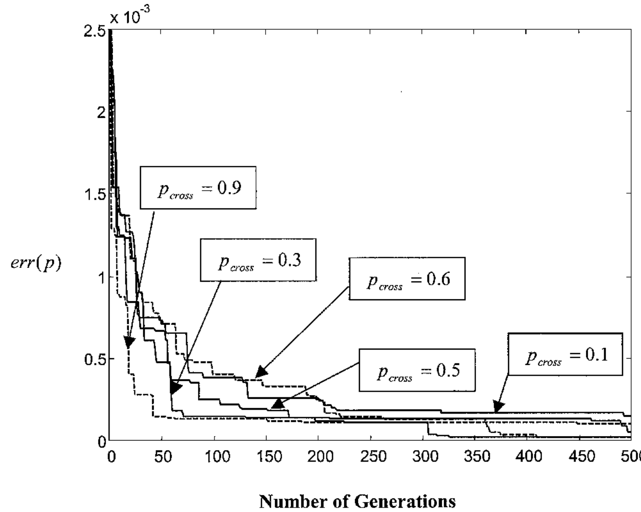

Fig. 3 Influence of the probability of the uniform crossover rate in the uniform μGA.

Parameters in Uniform μGA

A uniform μGA with binary parameter coding, tournament selection, uniform crossover, and elitism is adopted as the inverse operator. Knuth's subtractive method (see Refs. 35 and 36) is used to generate random numbers; Knuth's algorithm is regarded as one of the best random number generators. With a different negative number initialization, Knuth's algorithm generates different series of random number. An elitism operator is adopted to replicate the best individual of current generation into next generation. There is no mutation operation for the population evaluation in the uniform μGA. The population convergence criterion is 5%, which means when less than 5% of the total bits of other individuals in a generation are different than the best individual, convergence occurs. The stopping criterion is imposed to limit each GA run to a maximum number of generations.

Parameters, such as population size, chromosome string length, and probability of crossover, are chosen from the literature and numerical experiments. As recommended by Krishnakumar,[26] the population size of each generation is set to five; tournament selection and elitism are used. The value of integer numbers of possibilities for each parameter is set to 2^n because the uniform μGA has difficulty with parameters with long bit strings and non-2^n values. Because the probability of uniform crossover is the most important parameter in uniform μGA, it is studied in detail through a numerical experiment on the material characterization of SiC–C FGM. The influence of the uniform crossover p_{cross} rate on the performance of the uniform μGA is shown in Fig. 3. It appears that the probability of uniform crossover controls the solution convergence speed, but this relationship does not have regularity. The detailed result is shown in Table 3. For a smaller value of crossover rate, for example, $p_{\text{cross}} = 0.1$, the convergence is slow. For a bigger value of crossover rate, for example, $p_{\text{cross}} = 0.9$, the convergence is fast initially but slower at later stages. Compared to the case of $p_{\text{cross}} = 0.6$, the convergence for $p_{\text{cross}} = 0.5$ is faster. The final values of the fitness function are same for $p_{\text{cross}} = 0.5$ and $p_{\text{cross}} = 0.6$, but smaller than all other cases. From the maximum derivation of the individual parameter shown in Table 3, we have found the result is more accurate when $p_{\text{cross}} = 0.6$. Therefore, $p_{\text{cross}} = 0.6$ is used in this work.

Table 4 Characterized volume fractions of SiC–C FGM plate for different search ranges and noise levels of contamination

Position $\bar{z}$	Volume fraction	Original data	Results (deviation) for different search range 20%	30%	40%	50%
			Noise free			
0.0	V_P^1	1.000	1.000(0%)	1.000(0%)	1.000(0%)	1.000(0%)
0.2	V_P^2	0.961	0.966(0.5%)	0.959(−0.2%)	0.952(−0.9%)	0.969(0.8%)
0.4	V_P^3	0.842	0.835(−0.8%)	0.824(−2.0%)	0.896(6.4%)	0.809(−3.9%)
0.6	V_P^4	0.634	0.629(−0.8%)	0.665(4.9%)	0.604(4.7%)	0.532(−16%)
0.8	V_P^5	0.367	0.364(−0.8%)	0.361(1.6%)	0.421(15%)	0.314(14.4%)
1.0	V_P^6	0.000	0.008(−)	0.005(−)	0.009(−)	0.009(−)
			2% Noise			
0.0	V_P^1	1.000	1.000 (0%)	1.000 (0%)	1.000 (0%)	1.000 (0%)
0.2	V_P^2	0.961	0.954(−0.7%)	0.957(−0.4%)	0.963(0.2%)	0.960(−0.1%)
0.4	V_P^3	0.842	0.862(2.3%)	0.853(1.3%)	0.750(−11%)	0.806(−4.3%)
0.6	V_P^4	0.634	0.613(−3%)	0.601(−5%)	0.758(19.6%)	0.719(13.4%)
0.8	V_P^5	0.367	0.357(−2.7%)	0.390(6.3%)	0.365(−0.5%)	0.334(−9.0%)
1.0	V_P^6	0.000	0.006(−)	0.003(−)	0.007(−)	0.009(−)
			5% Noise			
0.0	V_P^1	1.000	1.000(0%)	1.000(0%)	0.999(0.1%)	1.000(0%)
0.2	V_P^2	0.961	0.956(−0.6%)	0.959(−0.2%)	0.993(3%)	0.934(2.8%)
0.4	V_P^3	0.842	0.859(2%)	0.812(3.6%)	0.886(5.2%)	0.961(14.1%)
0.6	V_P^4	0.634	0.605(−4.6%)	0.692(9.1%)	0.412(−35%)	0.394(−38%)
0.8	V_P^5	0.367	0.361(−1.6%)	0.335(−8.7%)	0.440(19.9%)	0.527(44%)
1.0	V_P^6	0.000	0.008(−)	0.006(−)	0.007(−)	0.008(−)
			10% Noise			
0.0	V_P^1	1.000	1.000(0%)	1.000(0%)	0.999(−0.1%)	1.000(0%)
0.2	V_P^2	0.961	0.952(−0.9%)	0.955(−0.6%)	0.995(0.6%)	0.950(−1%)
0.4	V_P^3	0.842	0.856(1.7%)	0.862(2.3%)	0.876(3.9%)	0.930(11%)
0.6	V_P^4	0.634	0.643(1.4%)	0.599(−5.5%)	0.445(−29.8%)	0.375(−41%)
0.8	V_P^5	0.367	0.336(8%)	0.395(7.6%)	0.402(9.5%)	0.533(45%)
1.0	V_P^6	0.000	0.008(−)	0.007(−)	0.005(−)	0.007(−)

Search Range

The bounds on the parameters are required to define a finite search space for the uniform μGA. Note that the volume fraction is usually controlled in the process of fabricating FGM; therefore, a rough distribution of the volume fraction is known when the FGM is fabricated. Hence, a range for the distribution of the volume fractions can be determined. In this work, ranges are assumed from ±20% to ±50% off from the actual value of the volume fraction.

Table 4 summarizes the characterized result for the SiC–C FGM plate. The volume fraction for carbon is to be characterized. It is seen that the accuracy of the characterization decreases as the search range increases, and the characterized results are very accurate when the search range is up to 30% from the actual value. It is also found that the error for the characterized volume fractions increase as the noise level increases. When the search range is smaller than ±30% off from the actual value, the characterized results remain stable regardless of the levels of noise, even for the level of 10%. It can be concluded that the present characterization procedure is very reliable if the search range is ±30% off from the actual value. The current fabricating technology can usually control the volume fraction at least within 20% of error. A search range of ±30% off from the actual value for the volume fractions should be robust enough. Note that the errors shown in Table 4 are not representative. We present these results only for a rough determination of the search range.

Results for SS–SN FGM Plate

The present inverse procedure is also applied to characterize the volume fractions for the SS–SN FGM plate. The plate is divided into six elements. It is assumed that this SS–SN FGM plate is made in such a way that the upper surface is pure SN, and the lower surface is pure SS. Therefore, the volume fractions on the upper and lower surfaces are known as 1.0 and 0.0, respectively. Hence, there are five parameters in total that need to be characterized. The original values of the volume fractions are assumed as the following function:

$$V_P = 1 - \bar{z}^3 \tag{18}$$

where $\bar{z} = z/H$. Based on the study carried out in SiC–C FGM plate the case, the bounds on the five parameters are set ±30% off from the actual value. The five volume fractions are described and translated into a chromosome of length 38. In the whole search space, there are a total of 2^{38} possible combinations of the five parameters. The search space for these parameters is listed in Table 5. The optimal parameters in the uniform μGA run for characterizing the material property of the SS–SN FGM plate are set as follows. The stopping criterion is imposed to limit each GA run to a maximum of 500 generations. The population size of each generation is set to 5. The optimal probability of uniform crossover is set to 0.6. The characterized results based on noise-free and Gauss noise-contaminated input data with a noise level of 2, 5, and 10% are listed in Table 6. The present inverse procedure gives very accurate results as shown in Table 6. Also note that the characterized results are very stable regardless of the levels of noise, even for the noise level of 10%.

It should be concluded that the present inverse procedure could be employed to characterize the material properties of FGMs using the measured displacement responses on the surface, provided the experiment error is within 10%.

Table 5 Uniform μGA search space for the material characterization of SS–SN FGM plate

Position $\bar{z}$	Volume fraction	Original data	Search range	Possibility number	Binary digit
0.3	V_P^1	0.973	0.681–1.000	256	8
0.5	V_P^2	0.875	0.613–1.000	256	8
0.7	V_P^3	0.657	0.456–0.854	256	8
0.8	V_P^4	0.488	0.342–0.634	128	7
0.9	V_P^5	0.271	0.100–0.352	128	7

Table 6 Characterized volume fractions for SS–SN FGM plate

Position noise $\bar{z}$	Volume fraction	Original data	Results (deviation) for different noise levels			
			Noise free	2% Noise	5% Noise	10% Noise
0.3	V_P^1	0.973	0.967(−0.6%)	0.974(0.1%)	0.976(0.3%)	0.976(0.3%)
0.5	V_P^2	0.875	0.860(−1.7%)	0.866(−1.0%)	0.852(−2.6%)	0.863(−1.4%)
0.7	V_P^3	0.657	0.656(−0.01%)	0.671(2.1%)	0.685(4.3%)	0.676(2.9%)
0.8	V_P^4	0.488	0.488(0%)	0.502(2.9%)	0.507(3.9%)	0.500(2.4%)
0.9	V_P^5	0.271	0.276(2.0%)	0.259(−4.3%)	0.260(−4.2%)	0.253(−8.6%)

In each uniform μGA run, the forward calculation subprogram is called for 2500 times; this shows the GA's high demand on the efficiency of the forward calculation. It has been proved in this work that the efficiency of the modified HNM made it possible to characterize material properties of FGM using GA from the displacement response on the surface.

Conclusions

An inverse procedure was successfully employed to obtain the material properties of FGMs from the displacement response on the surface. The modified HNM is used as a forward approach. A uniform μGA is adopted as the inverse operator controlling the forward solver. Material characterization using the present method is performed for two FGM systems. It has been found that the search range can be as large as $\pm 30\%$ off from the actual value of the volume fraction for characterizing material property of FGM plate. The numerical experiment suggested that the probability of the uniform crossover should be 0.6 in the uniform μGA. The present inverse process provides an accurate method to determine the material properties of FGMs from the measured displacements response on the surface. The error of the experiment can be as large as 10%.

All of the aspects illustrated in this paper are numerical calculation methods. The application of this proposed computational technique to practical engineering problems should be supported by experiment in further study. Furthermore, the presented technique is computationally expensive, and the GA performance at the later stage of searching is very slow compared to gradient-based methods. We can expect that the present technique could be improved by combining the advantages of both GA and gradient-based methods.

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